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A turbulence model for the pressure–strain correlation term accounting for compressibility effects

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Abstract

Compressibility effects on turbulence are examined with special focus on the intercomponent energy transfer via the pressure–strain term in compressible turbulence. Experimental studies often show that the growth rate of compressible mixing layers is reduced with increasing Mach number. The recent analysis of the direct numerical simulation data bases for compressible turbulence shows that this reduction is due to the suppression of the pressure–strain correlation in the compressible mixing layer. In this paper, the order of magnitude analysis in compressible turbulence is performed to derive a turbulence model for the pressure–strain term in which this compressibility effect is included. The derived model is used to simulate compressible mixing layers, showing that the model predicts the reduced growth rate observed in experimental studies.

Keywords: Compressible mixing layers; Turbulence model; Pressure–strain term

1. Introduction

Compressibility effects on turbulence have been studied extensively and numerical simulations of compressible flows using compressible turbulence models have been performed by Viegas and Rubesin (1990), Sarkar and Lakshmanan (1991) and Wilcox (1992). In view of the engineering importance, a compressible turbulence model is inevitable for numerical simulation of high-speed mixing layers. It is well known that the growth rate of high-speed mixing layers is critically reduced with increasing convective Mach number (for example Papamoschou and Roshka, 1988 and the compiled data as a test case for the 1980–1981 AFOSR–HTTM–STANFORD conference (Kline et al., 1981)). Bradshaw (1977) showed that the reduced growth rate is due to the effect of compressibility on turbulence. Direct numerical simulations (DNSs) of compressible turbulence have been performed to clarify the compressibility effect with special focus on the dilatational terms such as the dilatation dissipation and the pressure–dilatation correlation (for example Blaisdell et al., 1991). Yoshizawa et al. (1997) proposed a turbulence model, in which the compressibility effect on the turbulent viscosity is expressed in terms of the ratio of the normalized density variance to the squared turbulent Mach number. Speziale and Sarkar (1991) also proposed second-order closure models for supersonic turbulent flows. Turbulence models for the dilatational terms have been proposed by Sarkar et al. (1991), Sarkar (1992) and Zeman (1990, 1991). All of the above turbulence models perform well in predicting the reduced growth rate of the high-speed mixing layer.

On the other hand, recent work by Sarkar (1995) confirmed that the dilatational terms cannot be regarded as essential in causing the reduced growth rate. He also pointed out that compressibility was found to affect the production term more than the dissipation term. Vreman et al. (1996) concluded that reduced pressure fluctuations are responsible for the changes in the growth rate via the pressure–strain term. This conclusion is based on the examination of DNS data bases of compressible mixing layers. Recent experimental research on high-speed mixing layers performed by Goebel and Dutton (1993) shows that the anisotropy of the Reynolds stress tensor increases with increasing Mach number. Large eddy simulations of compressible mixing layers performed by Fujiwara et al. (1999) also show that the pressure–strain term decreases with increasing Mach number. This is an interesting phenomenon in view of turbulence modeling. The compressibility of turbulence affects the energy redistribution mechanism among the components of turbulence kinetic energy, which finally reduce the growth rate. A compressible turbulence model for the pressure–strain term was first proposed by Bonnet (1981) for the 1980–1981 AFOSR–HTTM–STANFORD conference (Kline et al., 1981), in which the slow part of the pressure–strain term is modified by including a fluctuating Mach number effect. A compressible turbulence model for the rapid part was proposed by El Baz and Launder (1993).

In this paper, the pressure–strain term is first divided into three parts: the root mean square of the pressure fluctuation, that of the strain rate fluctuation and the correlation coefficient

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between them. The reason for this division is that the pressure fluctuation, which is closely related to the compressibility of turbulence via the density fluctuation, can be treated explicitly. The order of magnitude analysis in compressible turbulence is performed to evaluate the three parts, showing that the relative magnitude of the pressure fluctuation to the turbulence kinetic energy should be reduced with increasing Mach number. This result agrees with the previous analyses of compressible turbulence performed by Vreman et al. (1996) and Goebel and Dutton (1993). Based on this result, a turbulence model for the pressure–strain term including the effect of compressibility is derived. The derived turbulence model is used to simulate the compressible mixing layer to check the performance of the model.

2. A turbulence model

Various turbulence models for the pressure–strain correlation have been proposed. The Rotta model (Rotta, 1951) and the isotropization of production model (IP model proposed by Launder et al. (1975)), are simplest and are still often used for practical simulation of compressible flows. The Rotta and IP models for compressible flows are expressed as

$$\Pi_{ij} = -C_1\overline{\rho}\epsilon\left(\frac{\overline{\rho u_i'' u_j''}}{\overline{\rho}k} - \frac{2}{3}\delta_{ij}\right) - C_2 P\left(\frac{P_{ij}}{P} - \frac{2}{3}\delta_{ij}\right), \quad (1)$$

$$= -C_1\,\overline{\rho}\epsilon\,\alpha_{ij} - C_2\,P\,\beta_{ij}, \quad (2)$$

$$\Pi_{ij} \equiv \overline{p' s_{ij}}, \quad (3)$$

$$P_{ij} \equiv -\overline{\rho u_j'' u_k''}\,\partial_k\widetilde{u}_i - \overline{\rho u_i'' u_k''}\,\partial_k\widetilde{u}_j, \quad P \equiv P_{kk}/2, \quad (4)$$

$$\overline{\rho}k \equiv \frac{1}{2}\,\overline{\rho u_i'' u_i''}, \quad (5)$$

$$\alpha_{ij} \equiv \frac{\overline{\rho u_i'' u_j''}}{\overline{\rho}k} - \frac{2}{3}\delta_{ij}, \quad \beta_{ij} \equiv \frac{P_{ij}}{P} - \frac{2}{3}\delta_{ij}, \quad (6)$$

$$s_{ij} \equiv u_{i,j}'' + u_{j,i}'', \quad (7)$$

where ϵ is the dissipation rate of turbulence kinetic energy k. The overbar denotes the conventional Reynolds average, while the overtilde is used to denote the Favre mass average. Double primes represent fluctuations with respect to the Favre average, while a single prime stands for fluctuations with respect to the Reynolds average.

The slow and rapid terms are modeled separately in the model (2). However, the pressure–strain term in compressible turbulence cannot be clearly divided into the rapid and slow parts due to the dilatation of velocity field. In the following analysis thereby a general form:

$$\Pi_{ij}/\overline{\rho}\epsilon = \left(C_1 + C_2\frac{P}{\overline{\rho}\epsilon}\right)F(\alpha_{ij}, \beta_{ij}, \ldots) \quad (8)$$

is used as a basis for including compressibility effect, where C_1 and C_2 are positive constants of order one and $F(\alpha_{ij}, \beta_{ij}, \ldots)$ is a function of anisotropy tensors whose value is also of order one. Before including compressibility effect, the model (8) is explained on the basis of the order of magnitude analysis for incompressible turbulence performed by Shikazono and Kasagi (1991, 1993) and Kasagi and Shikazono (1995).

The pressure–strain correlation can be written as a product of three terms: the root mean square of the pressure fluctuation, that of the strain rate fluctuation and the correlation coefficient between them:

$$\Pi_{ij} \equiv \overline{p' s_{ij}} = \sqrt{\overline{p'^2}}\sqrt{\overline{s_{ij}^2}}\,\underbrace{G(Re_T)F(\alpha_{ij}, \beta_{ij}, \ldots)}_{\text{(correlation coef.)}}. \quad (9)$$

The correlation coefficient is assumed to be a function of both the turbulent Reynolds number $Re_T(\equiv k^2/\nu\epsilon)$ and anisotropy tensors.

The magnitude of the strain rate fluctuation is approximated by

$$\sqrt{\overline{s_{ij}^2}} \sim \sqrt{\frac{\epsilon}{\nu}}, \quad (10)$$

while the magnitude of the pressure fluctuation is related to turbulence kinetic energy in low Mach number turbulence (see Shikazono and Kasagi, 1991, 1993):

$$\sqrt{\overline{p'^2}} \sim C_p\overline{\rho}k, \quad (11)$$

$$C_p = C_1 + C_2\frac{P}{\overline{\rho}\epsilon}. \quad (12)$$

The coefficient C_p is usually of order one in incompressible turbulence. Eq. (12) shows the phenomenon that C_p is larger in sheared turbulence than in decaying turbulence, which is often observed in DNSs of sheared and decaying turbulence.

Substituting Eqs. (10) and (11) into Eq. (9), we obtain

$$\Pi_{ij}/\overline{\rho}\epsilon = C_p\,\sqrt{Re_T}\,\underbrace{G(Re_T)\,F(\alpha_{ij}, \beta_{ij}, \ldots)}_{\text{(correlation coef.)}}. \quad (13)$$

If the correlation coefficient is O(1), the pressure–strain correlation is $\sqrt{Re_T}$ times as large as the dissipation rate. However, this is not the case because Re_T is usually much larger than unity in free shear turbulence. The pressure fluctuation is related to large eddies, while the strain rate fluctuation is related to small dissipative eddies. In high Reynolds number turbulence, the pressure and strain rate fluctuations cannot interact strongly because they are not tuned to the same frequency range. Tennekes and Lumley (1972) showed that the correlation coefficient should scale with the ratio of the time scales of these fluctuations, which is of order $Re_T^{-1/2}$:

$$G(Re_T) \to \frac{1}{\sqrt{Re_T}} \quad (Re_T \to \infty). \quad (14)$$

Applying relation (14) to Eq. (13), the general form is obtained

$$\Pi_{ij}/\overline{\rho}\epsilon = C_p\,F(\alpha_{ij}, \beta_{ij}, \ldots). \quad (15)$$

In the case of compressible turbulence, Eq. (11) should be revised. This can be explained as follows: the pressure in compressible turbulence is the thermodynamic pressure which is always positive:

$$p > 0. \quad (16)$$

This means that the pressure fluctuation should always satisfy the relation

$$p' > -\overline{p}. \quad (17)$$

Dividing both sides of Eq. (11) by $\overline{p}$, we have

$$\frac{\sqrt{\overline{p'^2}}}{\overline{p}} \sim C_p M_t^2, \quad (18)$$

$$M_t \equiv \frac{\sqrt{2k}}{\overline{a}}, \quad (19)$$

where M_t is the turbulent Mach number and $\overline{a}$ is the mean speed of sound. When $M_t \ll 1$, which corresponds to incompressible turbulence, the pressure fluctuation is much smaller than the mean value $\overline{p}$. On the other hand, the turbulent Mach number M_t may become nearly as large as unity in compressible turbulence. However, even in such compressible turbulence, $\sqrt{\overline{p'^2}}$ cannot become so large as $\overline{p}$ due to the strict limitation (17) that any pressure fluctuation can never be smaller than $-\overline{p}$. In other words, the eddy shocklets appear when the velocity fluctuations become as large as the mean speed of sound, which prevent the pressure fluctuations from becoming too large. In compressible turbulence, therefore, the coefficient C_p is a function of M_t, which should, at least, satisfy the condition

$$\begin{cases} C_p \sim 1 & (M_t \to 0), \\ C_p \to 0 & (M_t \to \infty). \end{cases} \tag{20}$$

The simplest way to express this compressibility effect is to multiply the RHS of Eq. (12) by a damping function $f(M_t)$:

$$C_p = f(M_t)\left(C_1 + C_2\frac{P}{\overline{\rho}\epsilon}\right). \tag{21}$$

Assuming that the LHS of Eq. (18), $\sqrt{\overline{p'^2}}/\overline{p}$, becomes asymptotically constant when $M_t \to \infty$, the damping function $f(M_t)$ should satisfy the condition

$$f(M_t) = \begin{cases} 1 & (M_t \to 0), \\ 1/M_t^2 & (M_t \to \infty). \end{cases} \tag{22}$$

For example, the following function satisfies the above condition (22):

$$f(M_t) = 1 - \exp(-C_f/M_t^2). \tag{23}$$

where C_f is a model constant. Finally, a turbulence model for the pressure–strain correlation accounting for the effect of compressibility is obtained:

$$\Pi_{ij}/\overline{\rho}\epsilon = f(M_t)\left(C_1 + C_2\frac{P}{\overline{\rho}\epsilon}\right)F(\alpha_{ij}, \beta_{ij}, \ldots). \tag{24}$$

Comparing this final form with Eq. (8), it can be said that the pressure–strain correlation model for compressible turbulence is obtained by multiplying the corresponding incompressible model by the damping function $f(M_t)$ which is always smaller than unity.

3. Application of the turbulence model

The above turbulence model was used to simulate compressible mixing layers. The governing equations are the Reynolds averaged Navier–Stokes equations with the additional equations for the Reynolds stresses and the turbulent dissipation rate (see Zha and Knight, 1996).

The equations for conservation of mass, momentum and energy are

$$\partial_t\overline{\rho} + \partial_k\overline{\rho}\widetilde{u}_k = 0, \tag{25}$$

$$\partial_t\overline{\rho}\widetilde{u}_i + \partial_k\overline{\rho}\widetilde{u}_i\widetilde{u}_k = -\partial_i\overline{p} + \partial_k\left(-\overline{\rho u_i'' u_k''} + \overline{\tau}_{ik}\right), \tag{26}$$

$$\begin{aligned}\partial_t\overline{\rho}\widetilde{e} + \partial_k(\overline{\rho}\widetilde{e} + \overline{p})\widetilde{u}_k &= \partial_k\left(-c_p\overline{\rho T'' u_k''} - \overline{q}_k\right) \\ &+ \partial_k\left(-\overline{\rho u_i'' u_k''}\widetilde{u}_i + \widetilde{u}_i\overline{\tau}_{ik}\right) + \partial_k\left(-\overline{\frac{1}{2}\rho u_j'' u_j'' u_k''} + \overline{u_i''\tau_{ik}}\right),\end{aligned} \tag{27}$$

$$\overline{\tau}_{ij} = -\frac{2}{3}\mu\partial_k\widetilde{u}_k\delta_{ij} + \mu(\partial_i\widetilde{u}_j + \partial_j\widetilde{u}_i), \tag{28}$$

$$\overline{q}_k = -\frac{c_p\mu}{Pr}\partial_k\widetilde{T}. \tag{29}$$

The following relations are assumed to evaluate $\overline{p}$ and $\widetilde{e}$:

$$\overline{p} = \overline{\rho}R\widetilde{T}, \tag{30}$$

$$\widetilde{e} = c_v\widetilde{T} + \frac{1}{2}\widetilde{u}_i\widetilde{u}_i + k, \tag{31}$$

where k is the turbulence kinetic energy defined by Eq. (5).

In the above relations, the Favre mass average is used for simplicity. Therefore the density–velocity correlation $\overline{\rho' u_i'}$ does not appear explicitly. Smits (1997) pointed out the density–velocity correlation cannot be neglected in hypersonic turbulent boundary layers. A turbulence model for the correlation has been already proposed by Yoshizawa (1992). One should note that such a correlation is not included in the following simulations. In the energy (27), the last term on the RHS is considered small and is neglected.

To close the above equations, a Reynolds stress transport equation model was used to determine the stress $-\overline{\rho u_i'' u_j''}$. The equation for the Reynolds stress is

$$\partial_t\overline{\rho u_i'' u_j''} + \partial_k\overline{\rho u_i'' u_j''}\widetilde{u}_k = P_{ij} + T_{ij} + \Pi_{ij} + D_{ij}, \tag{32}$$

$$P_{ij} = -\overline{\rho u_j'' u_k''}\partial_k\widetilde{u}_i - \overline{\rho u_i'' u_k''}\partial_k\widetilde{u}_j, \tag{33}$$

$$T_{ij} = \partial_k\left(-\overline{\rho u_i'' u_j'' u_k''} + \overline{u_j''\tau_{ik}} + \overline{u_i''\tau_{jk}} - \overline{p u_i''}\delta_{jk} - \overline{p u_j''}\delta_{ik}\right), \tag{34}$$

$$\Pi_{ij} = \overline{p'\left(\partial_j u_i'' + \partial_i u_j''\right)}, \tag{35}$$

$$D_{ij} = -\overline{\tau_{ik}\partial_k u_j''} - \overline{\tau_{jk}\partial_k u_i''}. \tag{36}$$

Several terms in the transport Eq. (32) should be modeled. Here, based on the LRR model proposed by Launder et al. (1975), we derive a simplified turbulence model which can be easily used in engineering application.

The diffusion term T_{ij} is simply modeled using the gradient diffusion hypothesis:

$$T_{ij} = \partial_k\left(\frac{\mu_t}{\sigma_k}\partial_k\overline{\rho u_i'' u_j''}\right), \tag{37}$$

where μ_t is the turbulent eddy viscosity:

$$\mu_t = C_\mu\frac{\overline{\rho}k^2}{\epsilon}. \tag{38}$$

The model for the pressure–strain correlation in compressible turbulence was derived in Eqs. (23) and (24) with the undefined function F of anisotropy tensors. The derivation of the precise and universal form of the function F is beyond the scope of this study. Instead, replacing the term $(C_1 + C_2P/\overline{\rho}\epsilon)F$ on the RHS of Eq. (24) by the RHS of the the Rotta and IP models Eq. (1), Π_{ij} is given by

$$\Pi_{ij} = f(M_t)\left\{-C_1\overline{\rho}\epsilon\left(\frac{\overline{\rho u_i'' u_j''}}{\overline{\rho}k} - \frac{2}{3}\delta_{ij}\right) - C_2P\left(\frac{P_{ij}}{P} - \frac{2}{3}\delta_{ij}\right)\right\}, \tag{39}$$

$$f(M_t) = 1 - \exp(-C_f/M_t^2). \tag{40}$$

One can see clearly that the present compressible model is obtained by multiplying the corresponding incompressible model by the dumpling function, as we have already explained in the previous section. The resultant Eq. (39) thereby shows that the damping rates on the slow (first) and rapid (second) parts are assumed to be exactly the same. It should be noted that this is one of the most important assumptions used to derive this equation. This relation is easy to use in engineering applications; however, it needs to be verified by using DNS data bases of compressible sheared turbulence.

The dissipation term D_{ij} is determined by using an isotropic dissipation model:

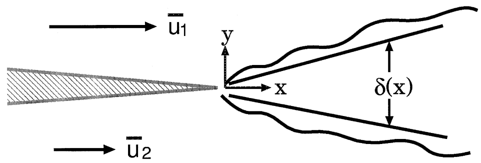

Fig. 1. The compressible turbulent mixing layer.

$$D_{ij} = -\frac{2}{3}\overline{\rho}\epsilon\delta_{ij}, \tag{41}$$

where ϵ is the dissipation rate of turbulence kinetic energy k. The dilatation dissipation and the pressure–dilatation correlation are not included in order to evaluate the performance of the present model clearly. The dissipation rate ϵ is obtained by solving the following transport equation:

$$\partial_t\overline{\rho}\epsilon + \partial_k\overline{\rho}\epsilon\widetilde{u}_k = C_{\epsilon 1}P\frac{\epsilon}{k} - C_{\epsilon 2}\frac{\overline{\rho}\epsilon^2}{k} + \partial_k\left\{\left(\mu + \frac{\mu_t}{\sigma_\epsilon}\right)\partial_k\epsilon\right\}. \tag{42}$$

The turbulent heat flux is modeled using a gradient diffusion hypothesis:

$$-c_p\overline{\rho T''u_i''} = c_p\frac{\mu_t}{Pr_T}\partial_i\widetilde{T}, \tag{43}$$

where Pr_T is the turbulent Prandtl number which is assumed constant.

The values of the constants used in the following simulation are

$$C_{\epsilon 1} = 1.4, \quad C_{\epsilon 2} = 1.8, \quad C_\mu = 0.09, \quad \sigma_k = 1.0, \quad \sigma_\epsilon = 1.4, \tag{44}$$

$$C_1 = 1.8, \quad C_2 = 0.6, \quad C_f = 0.02, \quad Pr_T = 0.8. \tag{45}$$

Note that the wall effect is not included in this model since only free shear flows are considered.

The above relations are used for the 2D simulation of the compressible mixing layer (Fig. 1). The inlet free stream conditions for velocity, density, static pressure and the mean speed of sound are

$$\overline{u}_1 = 2\,\overline{u}_2, \tag{46}$$

$$\overline{\rho}_1 = \overline{\rho}_2, \tag{47}$$

$$\overline{p}_1 = \overline{p}_2, \tag{48}$$

$$\overline{a}_1 = \overline{a}_2. \tag{49}$$

The subscript 1 denotes the value of the upper stream while the subscript 2 denotes the value of the lower stream. Fifteen cases are simulated with the convective Mach number changing from 0 to 2.0. The convective Mach number is defined by

$$M_c \equiv \frac{\overline{u}_1 - \overline{u}_2}{\overline{a}_1 + \overline{a}_2}. \tag{50}$$

The growth rate $d\delta/dx$ is calculated in each simulation. The width of the mixing layer δ is defined by the transverse distance between the two positions where the mean velocities are equal to $\overline{u}_2 + 0.1(\overline{u}_1 - \overline{u}_2)$ and $\overline{u}_2 + 0.9(\overline{u}_1 - \overline{u}_2)$. The calculated growth rates are normalized by the value of the incompressible case:

$$G \equiv \frac{d\delta/dx}{(d\delta/dx)_{Mc=0}} \tag{51}$$

and are compared with the experimental results obtained by Papamoschou and Roshka (1988) and Kline et al. (1981) in Fig. 2. The calculated growth rate decreases drastically with increasing convective Mach number. This phenomenon has often been observed in experimental studies of compressible mixing layers.

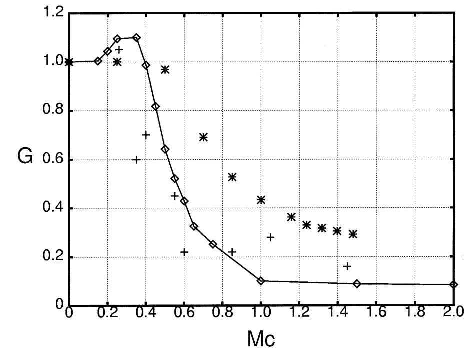

Fig. 2. Normalized growth rate G as a function of the convective Mach number M_c: (—◇—) current simulation, (+) Papamoschou and Roshka (1988), (∗) Kline et al. (1981).

4. Summary and future studies

The order of magnitude analysis in compressible turbulence shows that the relative magnitude of the pressure fluctuation to the turbulence kinetic energy should be reduced with increasing Mach number. In order to include this compressibility effect, a damping function $f(M_t)$ is introduced into a model for the pressure–strain correlation. The turbulence model is applied to the simulation of the compressible mixing layer, showing that the growth rate decreases with increasing convective Mach number, which is often observed in experimental studies.

For future studies, the wall effect should be taken into account in the present model. In that case, the pressure–strain correlation is suppressed due to the wall effect as well as the compressibility effect. Huang et al. (1995) showed that the compressibility effect in the compressible turbulent boundary layer is fairly small. This implies that the model constant C_f and/or the damping function $f(M_t)$ should be modified in the near–wall region.

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